Geometry and topology of parametric curves

Thématique: Algorithms, Complexity

Laboratoire, institution et université Inria & Sorbonne Université (LIP6 - Paris 6)

Ville et Pays Paris, France

Équipe-projets PolSys

Responsable: Elias Tsigaridas (elias.tsigaridas@inria.fr)

General description.

Let $\mathcal{C}$ be an algebraic curve in $\mathbb{C}^n$, that is the one-dimensional set of solutions of a system of polynomial equations in n variables with complex coefficients.

We call $\mathcal{C}$ a *rational curve* if there is a map

$$\psi : t \mapsto \left(\frac{f_1(t)}{g_1(t)}, \ldots, \frac{f_n(t)}{g_n(t)} \right),$$

where the f_i's and the g_i's are univariate polynomials and $t \in \mathbb{C}$, so that $\mathcal{C}$ is the Zariski closure of the image of ψ. The map ψ is the rational parametrization of $\mathcal{C}$.

There are a lot of results and algorithms concerning the geometry and the topology of curves in the real plane when they are given in implicit form, that is when the curve is given as the real zero-set of a bivariate polynomial $f(x, y) = 0$. However, very little is known for the topology and the geometry of rational curves $\mathcal{C} \cap \mathbb{R}^n$, with some notable exceptions, e.g., [1].

Objectives.

The objective of the internship is to survey carefully the known tools [2,3,4] for characterizing the (real) singularities of parametric curves in $\mathbb{R}^n$, to develop an algorithm for computing the topology, analyse its complexity, and present a prototype implementation in maple.

Other information. Interested candidates are encouraged to contact me for additional information.

Bibliography

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3. Andradas, Carlos, and Tomás Recio. “Plotting missing points and branches of real parametric curves.” Applicable Algebra in Engineering, Communication and Computing 18, no. 1-2 (2007): 107-126.

4. Manocha, D., Canny, J.F., 1992. “Detecting cusps and inflection points in curves.” Comput. Aided Geom. Design 9, 1–24.