Sparse Gröbner Bases: algorithms and complexity.

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Abstract

Toric (or sparse) elimination theory is a framework developed during the last decades to exploit monomial structures in systems of Laurent polynomials. We describe sparse variants of the F5 and FGLM algorithms that allow us to compute efficiently sparse Gröbner bases of sparse systems.

In the case where the generating subset of monomials corresponds to the points with integer coordinates in a normal lattice polytope and under some regularity assumptions, we prove complexity bounds which depend only on the combinatorial properties of the polytope. These bounds yield new estimates on the complexity of solving zero-dimensional systems where all polynomials share the same Newton polytope. For instance, we generalize the bound $\min(n_1, n_2)+2$ on the maximal degree in a Gröbner basis of a zero-dim. bilinear system with blocks of variables of sizes (n_1, n_2) to the multi-homogeneous case: for polynomials of multi-degree $(d_{1,}d_2, \ldots)$ w.r.t. a partition of the variables in blocks of sizes $(n_1, n_2, \ldots)$ the maximal degree occurring in a Gröbner basis computation is bounded by $2+\sum n_i - \max(\left\lceil \frac{n_i+1}{d_i} \right\rceil)$.

From a practical point of view, a proof-of-concept implementation shows large speed-ups compared to optimized Gröbner bases implementation.